Anonymity Networks

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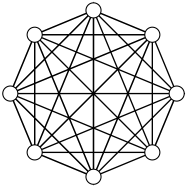
In the lecture *'Information Theory and Statistical Physics'* by Prof. Dr. Johannes Berg

motivation

- hide initiator of a message in a computer network
- safe whistleblowing under corporate and state surveillance
- ‘deniable communication’
- decentralized

idea



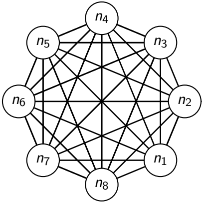
node network participant

link possible message path

- all nodes have equal weight
- message unmodifiable, only receiver is known
- each node on path: biased coin flip: forward or deliver
- each node on path: initiator or just forwarder?

→ message initator gets lost in the crowd

model



- N nodes $n_1, \ldots, n_N$ with $\mathbb{P}(n_i \text{ is initiator}) =: \mathbb{P}(X = n_i) =: p_i$
- n_i probably innocent $\leftrightarrow p_i \leq \frac{1}{2}$
- forwarding probability λ

if message received **then**
    flip biased coin $\mathbb{P}(\text{heads}) = \lambda$
    if heads **then**
        forward to a uniformly chosen node
    else
        deliver to receiver
    end if
end if

degree of anonymity

best case $\overline{X} := X : \forall i \in \{1, \ldots, N\} : p_i = \frac{1}{N}$

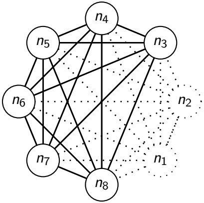
$$\overline{H} := H(\overline{X}) = -\sum_{i=1}^{N} p_i \cdot \ln(p_i) = \ln(N - C)$$

worst case $\underline{X} := X : \forall i \in \{1, \ldots, N\} \setminus \{j\} : p_i = 0 \wedge p_j = 1$

$$\underline{H} := H(\underline{X}) = -\sum_{i=1}^{N} p_i \cdot \ln(p_i) = 1 \cdot \ln(1) = 0$$

$$\boxed{d(X) := 1 - \frac{\overline{H} - H(X)}{\overline{H}} = \frac{H(X)}{\overline{H}} \in [0, 1]}$$

corruption

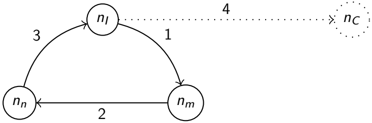


- $0 \leq C < N$ corrupt nodes (incoming message passer known)
- behave normally
- wait for message to be passed to us
- analyze probability of passer being initiator

$\mathbb{P}(\text{passer is initiator}) > \frac{1}{2} \rightarrow$ unmasked

analysis

events



let $k > 0$

$$H_k := \text{first corrupt node is at the } k\text{th path-position}$$

$$H_{k+} := \bigvee_{i=k}^{\infty} H_i$$

$$I := \text{first corrupt node immediately postcedes the message initiator}$$

$$\boxed{\mathbb{P}(\text{passer is initiator}) = \mathbb{P}(I | H_{1+})}$$

note: $H_1 \rightarrow I$, but $I \not\rightarrow H_1$

analysis

general probability I

$$\boxed{\mathbb{P}(I|H_{1+}) = \frac{N - \lambda(N - C - 1)}{N}}$$

proof:

$$\mathbb{P}(H_k) = \left(\lambda \cdot \frac{N - C}{N}\right)^{k-1} \cdot \left(\lambda \cdot \frac{C}{N}\right)$$

$$\Rightarrow \mathbb{P}(H_{k+}) = \sum_{i=k}^{\infty} \mathbb{P}(H_i) = \ldots = \frac{C \cdot \left(\lambda \cdot \frac{N-C}{N}\right)^k}{(N - C) \cdot \left(1 - \lambda \cdot \frac{N-C}{N}\right)}$$

$$H_1 \to I \Rightarrow \mathbb{P}(I|H_1) = 1$$

$$\mathbb{P}(I|H_{2+}) = \frac{1}{N - C}$$

analysis

general probability II

$$\mathbb{P}(I) \overset{TP}{=} \mathbb{P}(H_1)\,\mathbb{P}(I|H_1) + \mathbb{P}(H_{2+})\,\mathbb{P}(I|H_{2+}) = \dots$$

$$= \frac{\lambda \cdot C}{N} \cdot \left(1 + \frac{\lambda}{N - \lambda \cdot (N - C)}\right)$$

$$\mathbb{P}(I|H_{1+}) \overset{CP}{=} \frac{\mathbb{P}(I \wedge H_{1+})}{\mathbb{P}(H_{1+})} \quad \Big| \quad I \to H_{1+}$$

$$= \frac{\mathbb{P}(I)}{\mathbb{P}(H_{1+})} = \dots$$

$$= \frac{N - \lambda(N - C - 1)}{N} \qquad \square$$

good node $\mathbb{P}(\text{good node i is initiator}) = \frac{1-\mathbb{P}(I|H_{1+})}{N-C-1} = \frac{\lambda}{N} < \frac{1}{N} \leq \frac{1}{2}$

$\Rightarrow$ all good nodes besides passer are innocent

corrupt node $\mathbb{P}(\text{corrupt node i is initiator}) = 0$

$$\boxed{\text{passer innocent} \Leftrightarrow \lambda > \frac{1}{2} \wedge N \geq \frac{1}{1 - \frac{1}{2 \cdot \lambda}} \cdot (C + 1)}$$

proof:

$$\frac{1}{2} \geq \mathbb{P}(I|H_{1+}) = \frac{N - \lambda(N - C - 1)}{N} \quad \Bigg| \quad (\lambda - \frac{1}{2}) > 0$$

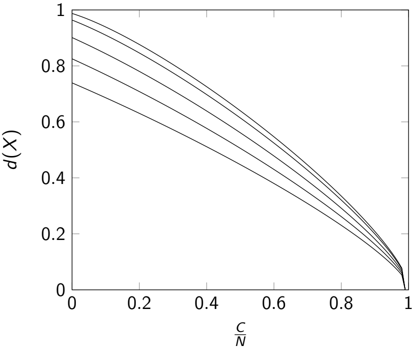
$$\Leftrightarrow N \geq \frac{1}{1 - \frac{1}{2 \cdot \lambda}} \cdot (C + 1) \qquad \square$$

analysis

degree of anonymity

$$d(X) = -\frac{C \cdot 0 + \mathbb{P}(I|H_{1+}) \cdot \ln(\mathbb{P}(I|H_{1+})) + (N - C - 1) \cdot \frac{\lambda}{N} \cdot \ln\left(\frac{\lambda}{N}\right)}{\ln(N - C)} =$$

$$= \ldots = \frac{(N-\lambda\cdot(N-C-1))\cdot\ln\left(\frac{N}{N-\lambda\cdot(N-C-1)}\right)+\lambda\cdot(N-C-1)\cdot\ln\left(\frac{N}{\lambda}\right)}{N\cdot\ln(N-C)}$$



conclusion

- blending in with the crowd works as long as it is large enough
- nothing to hide, but others to protect
- full paper on `http://frign.de/`