Approaching NSP-EMD with B-Splines

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Mathematisches Institut
Universität zu Köln

hosted by

Prof. Dr. Angela Kunoth

7th June 2018

introduction

intrinsic mode function (IMF) (1/2)

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- Physical reasonability.

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- Physical reasonability.
- Slowly varying amplitude.

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heuristic

- Physical reasonability.
- Slowly varying amplitude.
- Slowly varying frequency.

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intrinsic mode function (IMF) (2/2)

example

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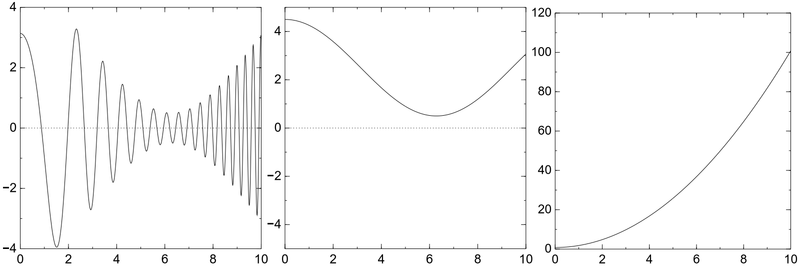

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empirical mode decomposition (EMD) (N.E. Huang et al., 1998)

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Find a_k and ϕ_k.

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Let $P_k = P_k(a_k, \phi_k)$, $Q_k = Q_k(a_k, \phi_k)$, $R_k = R_k(a_k, \phi_k)$.
Find differential operator $\mathcal{D}_{P_k,Q_k,R_k}$ analytically such that

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Solve

$$\begin{aligned}(P_k, Q_k, R_k) = \arg \min_{(\tilde{P},\tilde{Q},\tilde{R})} \quad & \|\mathcal{Q}_1(r_k - \tilde{s})\| \\ \text{s.t.} \quad & \mathcal{D}_{\tilde{P},\tilde{Q},\tilde{R}} \tilde{s} = 0 \\ & \|\mathcal{Q}_2 \tilde{P}\| \leq \tau \\ & \|\mathcal{Q}_3 \tilde{Q}\| \leq \tau \\ & \|\mathcal{Q}_4 \tilde{R}\| \leq \tau \end{aligned}$$

with $\tau > 0$, $\mathcal{Q}_1, \mathcal{Q}_2, \mathcal{Q}_3, \mathcal{Q}_4 \in \{D^0, D^1, \dots\}$ regularization operators, $\tilde{P} := P_k(\tilde{a}, \tilde{\phi})$, $\tilde{Q} := Q_k(\tilde{a}, \tilde{\phi})$, $\tilde{R} := R_k(\tilde{a}, \tilde{\phi})$ and $\tilde{s} := \tilde{a}(t) cos(\tilde{\phi}(t))$.

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unconstrained formulation

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(P_k, Q_k, R_k) = \arg \min_{(\tilde{P},\tilde{Q},\tilde{R})} & \|\mathcal{D}_{\tilde{P},\tilde{Q},\tilde{R}}\tilde{s}\| + \\
& \lambda \|\mathcal{Q}_1(r_k - \tilde{s})\| + \\
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Want to control $\tilde{s}$, and thus s_k, in some way.

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null space pursuit (NSP) (S. Peng, W.-L. Hwang et al., 2010)

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Introduce leakage factor γ.

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Controls retainment of information in residual signal.

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Does it really help understanding EMD?

vanishing operators

example (X. Hu, S. Peng, W.-L. Hwang, 2012)

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\mathcal{D}_{P_k, Q_k, R_k} &= \frac{d^2}{dt^2} + P_k(t)\frac{d}{dt} + Q_k(t).
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Let $a_k''(t) \neq 0$.

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Let $a_k''(t) \neq 0$. It holds

$$\mathcal{D}_{P_k,Q_k,R_k} s_k(t) = \frac{a_k''(t)}{a_k(t)} \neq 0.$$

vanishing operators

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Let $A_k(a_k) := \frac{a_k'}{a_k}$, $\Omega_k(\phi_k) := \frac{1}{(\phi_k')^2}$.

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Can we find a simple general form?

B-Splines

approach

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approach

- Express functions, e.g. the signal, as B-splines of order $k \geq 1$:

$$s(t) = \sum_{i=0}^{n-1} s_i B_{i,k}(t) \in \mathcal{C}^{k-2}.$$

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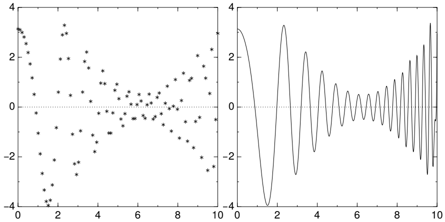
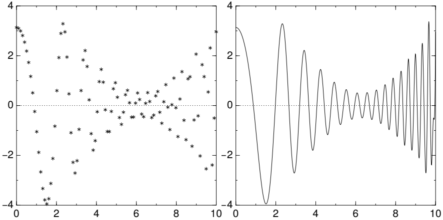


B-Splines
approach

- Express functions, e.g. the signal, as B-splines of order $k \geq 1$:

$$s(t) = \sum_{i=0}^{n-1} s_i B_{i,k}(t) \in \mathcal{C}^{k-2}.$$

- Least-Squares-Fit of time series data.



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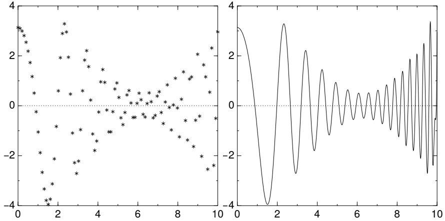

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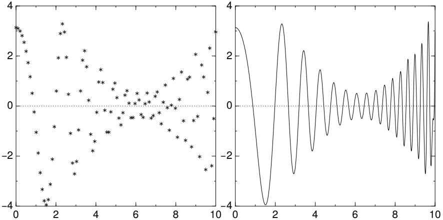

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- Directly optimize over (a_k, ϕ_k).

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- Constant time evaluation of $s_k^{(m)}$ only with a_k and ϕ_k using LEIBNIZ' rule and FAÀ DI BRUNO's formula after preprocessing multinomials and partitions.

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We can do better than that.

proposed optimization problem

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$$
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\text{s.t.} \quad & \tilde{a}(t) > 0 \\
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We can e.g. set $c(\tilde{a}, \tilde{\phi}) = \|\mathcal{Q}(r_k - \tilde{s})\|$ with $Q \in \{D^0, \ldots, D^{k-2}\}$.

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- Implement optimization problem.
- Investigate other cost functions.

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- Compare with other approaches.